3. How computers work
4. How to use a computer
5. The impact of computers on society
    a. Applications
    b. Occupations
    c. Uses and abuses of information gathering
6. How computers can develop the skills of decision making and coping with change
7. An introduction to programming
8. The history of computers

The computer was developed by humans to serve humans. Understanding the computer is the key to reducing the anxiety that accompanies this new technology. A well-organized, well-designed computer literacy course can provide a means to accomplish that goal.

STUDYING ON THE "LINE AT INFINITY"

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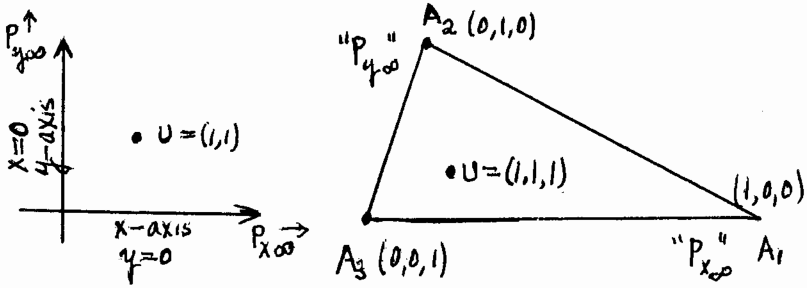
Sometimes it is instructive to look at things "another way" and different approaches often reveal properties new to us. The use of homogeneous coordinates does this and such a topic might well become a special study for students in upper high school mathematics.

Homogeneous coordinates in the X,Y-plane arise from replacing variables x and y by the <u>ratios</u> x_1/x_3 and x_2/x_3 respectively. New consequences follow immediately for the line whose equation is $3x + 2y = 10$ is now written as $3x_1 + 2x_2 = 10x_3$ or $3x_1 + 2x_2 - 10x_3 = 0$; the parabola $y = 3x^2 + 5$ becomes $x_2x_3 = 3x_1^2 + 5x_3^2$ and we note that the equations have become "homogeneous" because each term now has the same degree. The point (2,5) has its coordinates replaced by ratios 2/1 and 5/1 and we now write (2,5,1). The origin (0,0) becomes (0,0,1) while the unit point (1,1) assumes the name (1,1,1). Although the triple (x_1, x_2, x_3) is indeterminate if $x_1=0, x_2=0$ and $x_3=0$ simultaneously, the point $(a,b,0)$--where $a \neq b \neq 0$--represents a point on the "line at infinity" on the coordinate plane.

The coordinate axis system changes, too. Below are shown



the usual cartesian axes as well as the new homogeneous coordinate "reference triangle." With the use of homogeneous coordinates one can "bring into view" the points at infinity and treat them as any other point. The "point at infinity" on the old x-axis--now called the x_1-axis along which $x_2=0$--has coordinates $(1,0,0)$ and that for the old y-axis--now the x_2-axis along which $x_1=0$--has coodinates $(0,1,0)$. Every point on the line whose equation is $x_3 = 0$ is a "point at infinity" and the line is the "line at infinity" on the coordinate plane. On the non-homogeneous coordinate plane the equation of the line through the origin and the unit point $(1,1)$ is $y = x$, but in homogeneous coordinates it is $x_2/x_3 = x_1/x_3$ or $x_2 = x_1$ or $x_1 - x_2 = 0$. This equation can also be computed by the use of the standard method

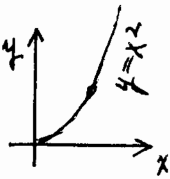
$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ x_1 & x_2 & x_3 \end{vmatrix} = 0.$$

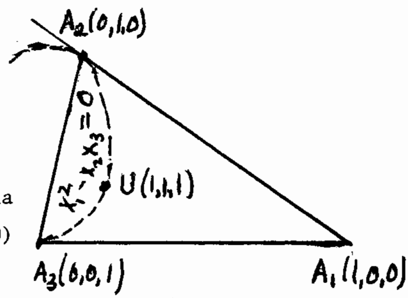
We note that the line $x_1 - x_2 = 0$ intersects the line at infinity, $x_3 = 0$, in a point whose coordinates satisfy (of course) the two equations simultaneously and this point has coordinates (1,1,0). The line A_2U has the equation given by

$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ x_1 & x_2 & x_3 \end{vmatrix} = 0 \quad \text{or} \quad x_1 - x_3 = 0$$

and it intersects the old x-axis (or $x_2 = 0$) in the point (1,0,1). Equations of other lines and coordinates of other interesting points can be found--all of which provide strength to the student.

The problem which prompts this discussion, however, concerns the parabola. As $x \to \infty$ then $y \to \infty$ and the question is, "How is the parabola related to L_∞ ?" Homogeneous coordinates will help. In the new system the equation $y = x^2$ becomes $x_2\, x_3 = x_1^2$ or $x_1^2 - x_2\, x_3 = 0$. The coordinates at (0,0,1) satisfy $x_1^2 - x_2 x_3 = 0$ so the parabola, as we would expect, goes through the origin. The coordinates of the point (0,1,0), or the "point at infinity" on the "y-axis" satisfy the equation of the parabola so it goes through A_2. Likewise, it is seen that the parabola goes through (1,1,1). As x becomes infinite and as y becomes infinite does the parabola intersect $x_3 = 0$ at $A_1 = (1,0,0)$ or at the "infinite end" of the x-axis? The answer is "No," and this is readily seen by noting that (1,0,0) does not satisfy the equation of the parabola.





Further study by the use of parameters shows that the parabola is also <u>tangent</u> to L_∞ or $x_3 = 0$ at A_2, but it was not intended to develop this technique in this discussion. Such studies are inviting once the method has been developed.

Students may well consider the behavior of the parabola $y^2 = x$ and the other conics $2x^2 + y^2 = 1$, $x^2 + 2y^2 = 1$ and $x^2 - y^2 = 1$ relative to L_∞.

Homogeneous coordinates are used in the study of the more general projective geometry in which one does not have affine, similarity or euclidean properties. The above discussion refers to figures which are valid in affine geometry at least although one need not think of that as he introduces these concepts to students.

DON'T FORGET BASE-TWELVE

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Many mathematics textbooks present the idea of counting in number systems besides the standard system of base-ten. Unfortunately, these discussions usually concern bases less than ten and rarely cover bases over ten. As a result, the base-twelve system is often ignored. This is an extreme oversight since the base-twelve system can be very enlightening to elementary aged students' study of number systems.

The reasons for teaching a system based on twelve are obvious. We have a name for a group of twelve objects: a dozen. Children know that they can buy a dozen eggs, a dozen oranges, or a dozen doughnuts. They have an experiential understanding that a dozen represents a group of twelve objects. Likewise, we also have a name for a group of twelve-twelves: a gross. Since these terms are familiar to students, it is only natural and logical to build on this familiarity.

However, before you begin an exploration of base-twelve, a minor obstacle must be overcome. In the base-ten system of numeration there are just ten numerals (0,1,2,3,4,5,6,7,8,9). To